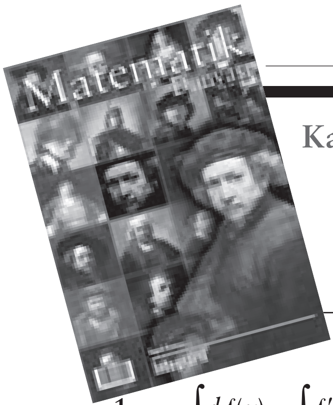

Kapak Konusu: İntegraller II

İntegral Formüleri

Bazı basit ifadeler

1. $\int d\, f(x) = \int f'(x)\, dx = f(x) + C$

2. $\int a\, du = a \int du$

3. $\int (du \pm dv \pm dw \pm \ldots) = \int du \pm \int dv \pm \int dw \pm \ldots$

4. $\int u^n\, du = \dfrac{u^{n+1}}{n+1} + C$

5. $\int \dfrac{du}{u} = \ln u + C$

$a + bu$ içeren rasyonel ifadeler

6. $\int (a+bu)^n\, du = \dfrac{(a+bu)^{n+1}}{b(n+1)} + C$

7. $\int \dfrac{du}{a+bu} = \dfrac{1}{b}\ln(a+bu) + C$

8. $\int \dfrac{u\, du}{a+bu} = \dfrac{1}{b^2}(a + bu - a\ln(a+bu)) + C$

9. $\int \dfrac{u^2 du}{a+bu} = \dfrac{1}{b^3}\left(\dfrac{1}{2}(a+bu)^2 - 2a(a+bu) + a^2\ln(a+bu)\right) + C$

10. $\int \dfrac{u\, du}{(a+bu)^2} = \dfrac{1}{b^2}\left(\dfrac{a}{a+bu} + \ln(a+bu)\right) + C$

11. $\int \dfrac{u^2 du}{(a+bu)^2} = \dfrac{1}{b^3}\left(a + bu - \dfrac{a^2}{a+bu} - 2a\ln(a+bu)\right) + C$

12. $\int \dfrac{u\, du}{(a+bu)^3} = \dfrac{1}{b^2}\left(-\dfrac{1}{a+bu} + \dfrac{a}{2(a+bu)^2}\right) + C$

13. $\int \dfrac{du}{u(a+bu)} = -\dfrac{1}{a}\ln\left(\dfrac{a+bu}{u}\right) + C$

14. $\int \dfrac{du}{u^2(a+bu)} = -\dfrac{1}{au} + \dfrac{b}{a^2}\ln\left(\dfrac{a+bu}{u}\right) + C$

15. $\int \dfrac{du}{u(a+bu)^2} = \dfrac{1}{a(a+bu)} - \dfrac{1}{a^2}\ln\left(\dfrac{a+bu}{u}\right) + C$

$a^2 \pm b^2u^2$ içeren ifadeler

16. $\int \dfrac{du}{a^2+b^2u^2} = \dfrac{1}{ab}\arctan\dfrac{bu}{a} + C$

17a. $\int \dfrac{du}{a^2-b^2u^2} = \dfrac{1}{2ab}\ln\left(\dfrac{a+bu}{a-bu}\right) + C \quad \left(a^2 > b^2u^2\right)$

17 b. $\displaystyle\int \frac{du}{b^2u^2-a^2} = \frac{1}{2ab}\ln\left(\frac{bu-a}{bu+a}\right)+C \quad \left(a^2<b^2u^2\right)$

18. $\displaystyle\int u\left(a^2 \pm b^2u^2\right)^n du = \frac{\left(a^2 \pm b^2u^2\right)^{n+1}}{\pm 2b^2(n+1)}+C \quad (n \neq -1)$

19. $\displaystyle\int \frac{udu}{a^2 \pm b^2u^2} = \frac{1}{\pm 2b^2}\ln\left(a^2 \pm b^2u^2\right)+C$

20. $\displaystyle\int \frac{u^m\,du}{\left(a^2 \pm b^2u^2\right)^p} = \frac{u^{m-1}}{\pm b^2(m-2p+1)\left(a^2 \pm b^2u^2\right)^{p-1}} - \frac{a^2(m-1)}{\pm b^2(m-2p+1)}\int \frac{u^{m-2}\,du}{\left(a^2 \pm b^2u^2\right)^p}$

21. $\displaystyle\int \frac{u^m\,du}{\left(a^2 \pm b^2u^2\right)^p} = \frac{u^{m+1}}{2a^2(p-1)(a^2 \pm b^2u^2)^{p-1}} - \frac{m-2p+3}{2a^2(p-1)}\int \frac{u^m du}{(a^2 \pm b^2u^2)^{p-1}}$

22. $\displaystyle\int \frac{du}{u\left(a^2 \pm b^2u^2\right)} = \frac{1}{2a^2}\ln\left(\frac{u^2}{a^2 \pm b^2u^2}\right)+C$

23. $\displaystyle\int \frac{du}{u^m\left(a^2 \pm b^2u^2\right)^p} = -\frac{1}{a^2(m-1)u^{m-1}\left(a^2 \pm b^2u^2\right)^{p-1}} - \frac{\pm b^2\left(m+2p-3\right)}{a^2(m-1)}\int \frac{du}{u^{m-2}\left(a^2 \pm b^2u^2\right)^p}$

24. $\displaystyle\int \frac{du}{u^m\left(a^2 \pm b^2u^2\right)^p} = \frac{1}{2a^2(p-1)u^{m-1}\left(a^2 \pm b^2u^2\right)^{p-1}} + \frac{m+2p-3}{2a^2(p-1)}\int \frac{du}{u^m\left(a^2 \pm b^2u^2\right)^{p-1}}$

$\sqrt{a+bu}$ içeren ifadeler

25. $\displaystyle\int u\sqrt{a+bu}\,du = -\frac{2(2a-3bu)(a+bu)^{\frac{3}{2}}}{15b^2}+C$

26. $\displaystyle\int u^2\sqrt{a+bu}\,du = -\frac{2(8a^2-12abu+15b^2u^2)(a+bu)^{\frac{3}{2}}}{105b^3}+C$

27. $\displaystyle\int u^m\sqrt{a+bu}\,du = \frac{2u^m(a+bu)^{\frac{3}{2}}}{b(2m+3)} - \frac{2am}{b(2m+3)}\int u^{m-1}\sqrt{a+bu}\,du$

28. $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = -\frac{2(2a-bu)\sqrt{a+bu}}{3b^2}+C$

29. $\displaystyle\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2\left(8a^2-4abu+3b^2u^2\right)\sqrt{a+bu}}{15b^3}+C$

30. $\displaystyle\int \frac{u^m\,du}{\sqrt{a+bu}} = \frac{2u^m\sqrt{a+bu}}{b(2m+1)} - \frac{2am}{b(2m+1)}\int \frac{u^{m-1}du}{\sqrt{a+bu}}$

31. $\displaystyle\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}}\ln\left(\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right)+C$ eğer $a>0$ ise

32. $\displaystyle\int \frac{du}{u\sqrt{a+bu}} = \frac{2}{\sqrt{-a}}\arctan\sqrt{\frac{a+bu}{-a}}+C$ eğer $a<0$ ise

33. $\displaystyle\int \frac{du}{u^m\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(m-1)u^{m-1}} - \frac{b(2m-3)}{2a(m-1)}\int \frac{du}{u^{m-1}\sqrt{a+bu}}$

34. $\displaystyle\int \frac{\sqrt{a+bu}\,du}{u} = 2\sqrt{a+bu} + a\int \frac{du}{u\sqrt{a+bu}}$

35. $\displaystyle\int \frac{\sqrt{a+bu}\,du}{u^m} = -\frac{(a+bu)^{\frac{3}{2}}}{a(m-1)u^{m-1}} - \frac{b(2m-5)}{2a(m-1)}\int \frac{\sqrt{a+bu}\,du}{u^{m-1}}$

$\sqrt{u^2 \pm a^2}$ içeren ifadeler

Bu formül grubunda

$\ln(\mathrm{u} + \sqrt{\mathrm{u}^2 + a^2})$ ifadesi $\arg\mathrm{sh}\frac{u}{a}$ ile

$\ln(\mathrm{u} + \sqrt{\mathrm{u}^2 - a^2})$ ifadesi $\arg\mathrm{ch}\frac{u}{a}$ ile

$\ln\frac{\mathrm{a} + \sqrt{\mathrm{u}^2 + a^2}}{u}$ ifadesi $\arg\mathrm{sh}\frac{a}{u}$ ile

değiştirilebilir.

36. $\displaystyle\int \left(u^2 \pm a^2\right)^{\frac{1}{2}} du = \frac{u}{2}\sqrt{u^2 \pm a^2} \pm \frac{a^2}{2}\ln\left(u + \sqrt{u^2 \pm a^2}\right) + C$

37. $\displaystyle\int \left(u^2 \pm a^2\right)^{\frac{n}{2}} du = \frac{u\left(u^2 \pm a^2\right)^{\frac{n}{2}}}{n+1} \pm \frac{na^2}{n+1}\int \left(u^2 \pm a^2\right)^{\frac{n}{2}-1} du \ (n \neq -1)$

38. $\displaystyle\int u\left(u^2 \pm a^2\right)^{\frac{n}{2}} du = \frac{\left(u^2 \pm a^2\right)^{\frac{n}{2}+1}}{n+2} + C \ \ (n \neq -2)$

39. $\displaystyle\int u^m\left(u^2 \pm a^2\right)^{\frac{n}{2}} du = \frac{u^{m-1}\left(u^2 \pm a^2\right)^{\frac{n}{2}+1}}{n+m+1} - \frac{\pm a^2(m-1)}{n+m+1}\int u^{m-2}\left(u^2 \pm a^2\right)^{\frac{n}{2}} du$

40. $\displaystyle\int \frac{du}{\left(u^2 \pm a^2\right)^{\frac{1}{2}}} = \ln\left(u + \sqrt{u^2 \pm a^2}\right) + C$

41. $\displaystyle\int \frac{du}{\left(u^2 \pm a^2\right)^{\frac{3}{2}}} = \frac{u}{\pm a^2\sqrt{u^2 \pm a^2}} + C$

42. $\displaystyle\int \frac{u\,du}{\left(u^2 \pm a^2\right)^{\frac{3}{2}}} = \frac{\left(u^2 \pm a^2\right)^{1-\frac{n}{2}}}{2-n} + C$

43. $\displaystyle\int \frac{u^2\,du}{\left(u^2 \pm a^2\right)^{\frac{1}{2}}} = \frac{u}{2}\sqrt{u^2 \pm a^2} - \frac{\pm a^2}{2}\ln\left(u + \sqrt{u^2 \pm a^2}\right) + C$

44. $\displaystyle\int \frac{u^2\,du}{\left(u^2 \pm a^2\right)^{\frac{n}{2}}} = -\frac{u}{\sqrt{u^2 \pm a^2}} + \ln\left(u + \sqrt{u^2 \pm a^2}\right) + C$

45. $\displaystyle\int \frac{u^m\,du}{\left(u^2 \pm a^2\right)^{\frac{n}{2}}} = -\frac{u^{m-1}}{(m-n+1)\left(u^2 \pm a^2\right)^{\frac{n}{2}-1}} - \frac{\pm a^2(m-1)}{m-n+1}\int \frac{u^{m-2}\,du}{\left(u^2 \pm a^2\right)^{\frac{n}{2}}}$

46. $$\int \frac{u^m\,du}{\left(u^2 \pm a^2\right)^{\frac{n}{2}}} = \frac{u^{m+1}}{\pm a^2(n-2)\left(u^2 \pm a^2\right)^{\frac{n}{2}-1}} - \frac{m-n+3}{\pm a^2(n-2)} \int \frac{u^m\,du}{\left(u^2 \pm a^2\right)^{\frac{n}{2}-1}}$$

47. $$\int \frac{du}{u\left(u^2 + a^2\right)^{\frac{1}{2}}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{u^2+a^2}}{u}\right) + C$$

48. $$\int \frac{du}{u\left(u^2 - a^2\right)^{\frac{1}{2}}} = \frac{1}{a}\operatorname{arcsec}\frac{u}{a} + C$$

49. $$\int \frac{du}{u^2\left(u^2 \pm a^2\right)^{\frac{1}{2}}} = -\frac{\sqrt{u^2 \pm a^2}}{\pm a^2 u} + C$$

50. $$\int \frac{du}{u^3\left(u^2 + a^2\right)^{\frac{1}{2}}} = -\frac{\sqrt{u^2+a^2}}{2a^2u^2} + \frac{1}{2a^3}\ln\left(\frac{a+\sqrt{u^2+a^2}}{u}\right) + C$$

51. $$\int \frac{du}{u^3\left(u^2 - a^2\right)^{\frac{1}{2}}} = \frac{\sqrt{u^2-a^2}}{2a^2u^2} + \frac{1}{2a^3}\operatorname{arcsec}\frac{u}{a} + C$$

52. $$\int \frac{du}{u^m\left(u^2 \pm a^2\right)^{\frac{n}{2}}} = -\frac{1}{\pm a^2(m-1)u^{m-1}\left(u^2 \pm a^2\right)^{\frac{n}{2}-1}} - \frac{m+n-3}{\pm a^2(m-1)} \int \frac{du}{u^{m-2}\left(u^2 \pm a^2\right)^{\frac{n}{2}}}$$

53. $$\int \frac{du}{u^m\left(u^2 \pm a^2\right)^{\frac{n}{2}}} = \frac{1}{\pm a^2(n-2)u^{m-1}\left(u^2 \pm a^2\right)^{\frac{n}{2}-1}} + \frac{m+n-3}{\pm a^2(n-2)} \int \frac{du}{u^m\left(u^2 \pm a^2\right)^{\frac{n}{2}-1}}$$

54. $$\int \frac{\left(u^2 \pm a^2\right)^{\frac{1}{2}}\,du}{u} = \sqrt{u^2+a^2} - a\ln\left(\frac{a+\sqrt{u^2+a^2}}{u}\right) + C$$

55. $$\int \frac{\left(u^2 - a^2\right)^{\frac{1}{2}}\,du}{u} = \sqrt{u^2-a^2} - a\operatorname{arcsec}\frac{u}{a} + C$$

56. $$\int \frac{\left(u^2 \pm a^2\right)^{\frac{1}{2}}\,du}{u^2} = -\frac{\sqrt{u^2 \pm a^2}}{u} + \ln\left(u + \sqrt{u^2 \pm a^2}\right) + C$$

57. $$\int \frac{\left(u^2 \pm a^2\right)^{\frac{n}{2}}\,du}{u^m} = -\frac{\left(u^2 \pm a^2\right)^{\frac{n}{2}+1}}{\pm a^2(m-1)u^{m-1}} - \frac{m-n-3}{\pm a^2(m-1)} \int \frac{\left(u^2 \pm a^2\right)^{\frac{n}{2}}\,du}{u^{m-2}}$$

58. $$\int \frac{\left(u^2 \pm a^2\right)^{\frac{n}{2}}\,du}{u^m} = \frac{\left(u^2 \pm a^2\right)^{\frac{n}{2}}}{(n-m+1)u^{m-1}} + \frac{\pm a^2 n}{n-m+1} \int \frac{\left(u^2 \pm a^2\right)^{\frac{n}{2}-1}\,du}{u^m}$$

$\sqrt{a^2-u^2}$ içeren ifadeler

59. $$\int \left(a^2 - u^2\right)^{\frac{1}{2}}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\arcsin\frac{u}{a} + C$$

60. $\int \left(a^2-u^2\right)^{\frac{n}{2}} du = \dfrac{u\left(a^2-u^2\right)^{\frac{n}{2}}}{n+1} + \dfrac{a^2 n}{n+1} \int \left(a^2-u^2\right)^{\frac{n}{2}-1} du \quad (n \neq -1)$

61. $\int u\left(a^2-u^2\right)^{\frac{n}{2}} du = -\dfrac{\left(a^2-u^2\right)^{\frac{n}{2}+1}}{n+2} + C \quad (n \neq -2)$

62. $\int u^m\left(a^2-u^2\right)^{\frac{n}{2}} du = -\dfrac{u^{m-1}\left(a^2-u^2\right)^{\frac{n}{2}+1}}{n+m+1} + \dfrac{a^2(m-1)}{n+m+1} \int u^{m-2}\left(a^2-u^2\right)^{\frac{n}{2}} du$

63. $\int \dfrac{du}{\left(a^2-u^2\right)^{\frac{1}{2}}} = \arcsin\dfrac{u}{a} + C$

64. $\int \dfrac{du}{\left(a^2-u^2\right)^{\frac{3}{2}}} = \dfrac{u}{a^2\sqrt{a^2-u^2}} + C$

65. $\int \dfrac{u\,du}{\left(a^2-u^2\right)^{\frac{n}{2}}} = \dfrac{\left(a^2-u^2\right)^{1-\frac{n}{2}}}{n-2} + C$

66. $\int \dfrac{u^2\,du}{\left(a^2-u^2\right)^{\frac{1}{2}}} = -\dfrac{u}{2}\sqrt{a^2-u^2} + \dfrac{a^2}{2}\arcsin\dfrac{u}{a} + C$

67. $\int \dfrac{u^2\,du}{\left(a^2-u^2\right)^{\frac{3}{2}}} = \dfrac{u}{\sqrt{a^2-u^2}} - \arcsin\dfrac{u}{a} + C$

68. $\int \dfrac{u^m\,du}{\left(a^2-u^2\right)^{\frac{n}{2}}} = -\dfrac{u^{m-1}}{(m-n+1)\left(a^2-u^2\right)^{\frac{n}{2}-1}} + \dfrac{a^2(m-1)}{m-n+1} \int \dfrac{u^{m-2}du}{\left(a^2-u^2\right)^{\frac{n}{2}}}$

69. $\int \dfrac{u^m\,du}{\left(a^2-u^2\right)^{\frac{n}{2}}} = \dfrac{u^{m+1}}{a^2(n-2)\left(a^2-u^2\right)^{\frac{n}{2}-1}} - \dfrac{m-n+3}{a^2(n-2)} \int \dfrac{u^m\,du}{\left(a^2-u^2\right)^{\frac{n}{2}-1}}$

70. $\int \dfrac{du}{u\left(a^2-u^2\right)^{\frac{1}{2}}} = -\dfrac{1}{a}\ln\left(\dfrac{a+\sqrt{a^2-u^2}}{u}\right) + C = -\dfrac{1}{a}\operatorname{arg\,coth}\dfrac{a}{u} + C$

71. $\int \dfrac{du}{u^2\left(a^2-u^2\right)^{\frac{1}{2}}} = -\dfrac{\sqrt{a^2-u^2}}{a^2 u} + C$

72. $\int \dfrac{du}{u^3(a^2-u^2)^{\frac{1}{2}}} = -\dfrac{\sqrt{a^2-u^2}}{2a^2u^2} - \dfrac{1}{2a^3}\ln\left(\dfrac{a+\sqrt{a^2-u^2}}{u}\right) + C = -\dfrac{\sqrt{a^2-u^2}}{2a^2u^2} - \dfrac{1}{2a^3}\operatorname{arg\,coth}\dfrac{a}{u} + C$

73. $\int \dfrac{du}{u^m\left(a^2-u^2\right)^{\frac{n}{2}}} = -\dfrac{1}{a^2(m-1)u^{m-1}\left(a^2-u^2\right)^{\frac{n}{2}-1}} + \dfrac{m+n-3}{a^2(m-1)} \int \dfrac{du}{u^{m-2}\left(a^2-u^2\right)^{\frac{n}{2}}}$

74. $$\int \frac{du}{u^m\left(a^2-u^2\right)^{\frac{n}{2}}} = \frac{1}{a^2(n-2)u^{m-1}\left(a^2-u^2\right)^{\frac{n}{2}-1}} + \frac{m+n-3}{a^2(n-2)}\int \frac{du}{u^m\left(a^2-u^2\right)^{\frac{n}{2}-1}}$$

75. $$\int \frac{\left(a^2-u^2\right)^{\frac{1}{2}}du}{u} = \sqrt{a^2-u^2} - a\ln\left(\frac{a+\sqrt{a^2-u^2}}{u}\right) + C = \sqrt{a^2-u^2} - a\,\mathrm{arg\,coth}\,\frac{a}{u} + C$$

76. $$\int \frac{\left(a^2-u^2\right)^{\frac{1}{2}}du}{u^2} = -\frac{\sqrt{a^2-u^2}}{u} - \arcsin\frac{u}{a} + C$$

77. $$\int \frac{\left(a^2-u^2\right)^{\frac{n}{2}}du}{u^m} = -\frac{\left(a^2-u^2\right)^{\frac{n}{2}+1}}{a^2(m-1)u^{m-1}} + \frac{m-n-3}{a^2(m-1)}\int \frac{\left(a^2-u^2\right)^{\frac{n}{2}}du}{u^{m-2}}$$

78. $$\int \frac{\left(a^2-u^2\right)^{\frac{n}{2}}du}{u^m} = \frac{\left(a^2-u^2\right)^{\frac{n}{2}}}{(n-m+1)u^{m-1}} + \frac{a^2 n}{n-m+1}\int \frac{\left(a^2-u^2\right)^{\frac{n}{2}-1}du}{u^m}$$

$\sqrt{2au \pm u^2}$ içeren ifadeler

$\sqrt{2au \pm u^2} = u^{1/2}(2a \pm u)^{1/2}$ eşitliği sayesinde 96-104 numaralı formüller de kullanılabilir.

79. $$\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\arccos\left(1-\frac{u}{a}\right) + C$$

80. $$\int u\sqrt{2au-u^2}\,du = -\frac{3a^2+au-2u^2}{6}\sqrt{2au-u^2} + \frac{a^3}{2}\arccos\left(1-\frac{u}{a}\right) + C$$

81. $$\int u^m\sqrt{2au-u^2}\,du = -\frac{u^{m-1}\left(2au-u^2\right)^{\frac{3}{2}}}{m+2} + \frac{a(2m+1)}{m+2}\int u^{m-1}\sqrt{2au-u^2}\,du$$

82. $$\int \frac{\sqrt{2au-u^2}\,du}{u} = \sqrt{2au-u^2} + a\arccos\left(1-\frac{u}{a}\right) + C$$

83. $$\int \frac{\sqrt{2au-u^2}\,du}{u^2} = -\frac{2\sqrt{2au-u^2}}{u} - \arccos\left(1-\frac{u}{a}\right) + C$$

84. $$\int \frac{\sqrt{2au-u^2}\,du}{u^3} = -\frac{\left(2au-u^2\right)^{\frac{3}{2}}}{3au^3} + C$$

85. $$\int \frac{\sqrt{2au-u^2}\,du}{u^m} = -\frac{\left(2au-u^2\right)^{\frac{3}{2}}}{a(2m-3)u^m} + \frac{m-3}{a(2m-3)}\int \frac{\sqrt{2au-u^2}\,du}{u^{m-1}}$$

86. $$\int \frac{du}{\sqrt{2au-u^2}} = \arccos\left(1-\frac{u}{a}\right) + C$$

87. $$\int \frac{du}{\sqrt{2au+u^2}} = \ln\left(u+a+\sqrt{2au+u^2}\right) + C$$

88. $$\int F\left(u,\sqrt{2au+u^2}\right)du = \int F\left(z-a,\sqrt{z^2-a^2}\right)dz \quad (\text{burada } z = u+a)$$

89. $$\int \frac{u\,du}{\sqrt{2au-u^2}} = -\sqrt{2au-u^2} + a\arccos\left(1-\frac{u}{a}\right) + C$$

90. $$\int \frac{u^2\,du}{\sqrt{2au-u^2}} = -\frac{(u+3a)\sqrt{2au-u^2}}{2} + \frac{3a^2}{2}\arccos\left(1-\frac{u}{a}\right)+C$$

91. $$\int \frac{u^m\,du}{\sqrt{2au-u^2}} = -\frac{u^{m-1}\sqrt{2au-u^2}}{m} + \frac{a(2m-1)}{m}\int \frac{u^{m-1}\,du}{\sqrt{2au-u^2}}$$

92. $$\int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au}+C$$

93. $$\int \frac{du}{u^m\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{a(2m-1)u^m} + \frac{m-1}{a(2m-1)}\int - \frac{du}{u^{m-1}\sqrt{2au-u^2}}$$

94. $$\int \frac{du}{\left(2au-u^2\right)^{\frac{3}{2}}} = \frac{u-a}{a^2\sqrt{2au-u^2}}+C$$

95. $$\int \frac{u\,du}{\left(2au-u^2\right)^{\frac{3}{2}}} = \frac{u}{a\sqrt{2au-u^2}}+C$$

Binomlar

96. $$\int u^m\left(a+bu^q\right)^p du = \frac{u^{m-q+1}\left(a+bu^q\right)^{p+1}}{b(pq+m+1)} - \frac{a(m-q+1)}{b(pq+m+1)}\int u^{m-q}\left(a+bu^q\right)^p du$$

97. $$\int u^m\left(a+bu^q\right)^p du = \frac{u^{m+1}\left(a+bu^q\right)^{p}}{pq+m+1} + \frac{apq}{pq+m+1}\int u^{m}\left(a+bu^q\right)^{p-1} du$$

98. $$\int \frac{du}{u^m\left(a+bu^q\right)^p} = -\frac{1}{a(m-1)u^{m-1}\left(a+bu^q\right)^{p-1}} - \frac{b(m-q+pq-1)}{a(m-1)}\int \frac{du}{u^{m-q}\left(a+bu^q\right)^p}$$

99. $$\int \frac{du}{u^m\left(a+bu^q\right)^p} = \frac{1}{aq(p-1)u^{m-1}\left(a+bu^q\right)^{p-1}} + \frac{m-q+pq-1}{aq(p-1)}\int \frac{du}{u^{m}\left(a+bu^q\right)^{p-1}}$$

100. $$\int \frac{du}{u\left(a+bu^q\right)} = \frac{1}{aq}\ln\left(\frac{u^q}{a+bu^q}\right)+C$$

101. $$\int \frac{\left(a+bu^q\right)^p du}{u^m} = -\frac{\left(a+bu^q\right)^{p+1}}{a(m-1)u^{m-1}} - \frac{b(m-q-pq-1)}{a(m-1)}\int \frac{\left(a+bu^q\right)^p du}{u^{m-q}}$$

102. $$\int \frac{\left(a+bu^q\right)^p du}{u^m} = \frac{\left(a+bu^q\right)^{p}}{(pq-m+1)u^{m-1}} + \frac{apq}{pq-m+1}\int \frac{\left(a+bu^q\right)^{p-1} du}{u^{m}}$$

103. $$\int \frac{u^m\,du}{\left(a+bu^q\right)} = \frac{u^{m-q+1}}{b(m-pq+1)\left(a+bu^q\right)^{p-1}} - \frac{a(m-q+1)}{b(m-pq+1)}\int \frac{u^{m-q}du}{\left(a+bu^q\right)^p}$$

104. $$\int \frac{u^m du}{\left(a+bu^q\right)^p} = \frac{u^{m+1}}{aq(p-1)\left(a+bu^q\right)^{p-1}} - \frac{m+q-pq+1}{aq(p-1)}\int \frac{u^m\,du}{\left(a+bu^q\right)^{p-1}}$$

90. $$\int \frac{u^2\,du}{\sqrt{2au-u^2}} = -\frac{(u+3a)\sqrt{2au-u^2}}{2} + \frac{3a^2}{2}\arccos\left(1-\frac{u}{a}\right) + C$$

91. $$\int \frac{u^m\,du}{\sqrt{2au-u^2}} = -\frac{u^{m-1}\sqrt{2au-u^2}}{m} + \frac{a(2m-1)}{m}\int \frac{u^{m-1}\,du}{\sqrt{2au-u^2}}$$

92. $$\int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au} + C$$

93. $$\int \frac{du}{u^m\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{a(2m-1)u^m} + \frac{m-1}{a(2m-1)}\int \frac{du}{u^{m-1}\sqrt{2au-u^2}}$$

94. $$\int \frac{du}{\left(2au-u^2\right)^{\frac{3}{2}}} = \frac{u-a}{a^2\sqrt{2au-u^2}} + C$$

95. $$\int \frac{u\,du}{\left(2au-u^2\right)^{\frac{3}{2}}} = \frac{u}{a\sqrt{2au-u^2}} + C$$

Binomlar

96. $$\int u^m\left(a+bu^q\right)^p du = \frac{u^{m-q+1}\left(a+bu^q\right)^{p+1}}{b(pq+m+1)} - \frac{a(m-q+1)}{b(pq+m+1)}\int u^{m-q}\left(a+bu^q\right)^p du$$

97. $$\int u^m\left(a+bu^q\right)^p du = \frac{u^{m+1}\left(a+bu^q\right)^p}{pq+m+1} + \frac{apq}{pq+m+1}\int u^m\left(a+bu^q\right)^{p-1} du$$

98. $$\int \frac{du}{u^m\left(a+bu^q\right)^p} = -\frac{1}{a(m-1)u^{m-1}\left(a+bu^q\right)^{p-1}} - \frac{b(m-q+pq-1)}{a(m-1)}\int \frac{du}{u^{m-q}\left(a+bu^q\right)^p}$$

99. $$\int \frac{du}{u^m\left(a+bu^q\right)^p} = \frac{1}{aq(p-1)u^{m-1}\left(a+bu^q\right)^{p-1}} + \frac{m-q+pq-1}{aq(p-1)}\int \frac{du}{u^m\left(a+bu^q\right)^{p-1}}$$

100. $$\int \frac{du}{u\left(a+bu^q\right)} = \frac{1}{aq}\ln\left(\frac{u^q}{a+bu^q}\right) + C$$

101. $$\int \frac{\left(a+bu^q\right)^p du}{u^m} = -\frac{\left(a+bu^q\right)^{p+1}}{a(m-1)u^{m-1}} - \frac{b(m-q-pq-1)}{a(m-1)}\int \frac{\left(a+bu^q\right)^p du}{u^{m-q}}$$

102. $$\int \frac{\left(a+bu^q\right)^p du}{u^m} = \frac{\left(a+bu^q\right)^p}{(pq-m+1)u^{m-1}} + \frac{apq}{pq-m+1}\int \frac{\left(a+bu^q\right)^{p-1} du}{u^m}$$

103. $$\int \frac{u^m\,du}{\left(a+bu^q\right)} = \frac{u^{m-q+1}}{b(m-pq+1)\left(a+bu^q\right)^{p-1}} - \frac{a(m-q+1)}{b(m-pq+1)}\int \frac{u^{m-q}du}{\left(a+bu^q\right)^p}$$

104. $$\int \frac{u^m du}{\left(a+bu^q\right)^p} = \frac{u^{m+1}}{aq(p-1)\left(a+bu^q\right)^{p-1}} - \frac{m+q-pq+1}{aq(p-1)}\int \frac{u^m\,du}{\left(a+bu^q\right)^{p-1}}$$

119. $\int \frac{du}{\sqrt{(u-a)(b-u)}} = 2\arcsin\sqrt{\frac{u-a}{b-a}} + C$

Eksp üstel ve logaritmik ifadeler

120. $\int e^{au}\, du = \frac{e^{au}}{a} + C$

121. $\int b^{au}\, du = \frac{b^{au}}{a\ln(b)} + C$

122. $\int ue^{au}\, du = \frac{e^{au}}{a^2}(au-1) + C$

123. $\int u^n e^{au}\, du = \frac{u^n e^{au}}{a} - \frac{n}{a}\int u^{n-1}e^{au}\, du$

124. $\int u^n b^{au}\, du = \frac{u^n b^{au}}{a\ln(b)} - \frac{n}{a\ln(b)}\int u^{n-1}b^{au}\, du$

125. $\int \frac{b^{au}}{u^n} = -\frac{b^{au}}{(n-1)u^{n-1}} + \frac{a\ln(b)}{n-1}\int \frac{b^{au}\, du}{u^{n-1}}$

126. $\int \ln(u)\, du = u\ln(u) - u + C$

127. $\int u^n \ln(u)\, du = u^{n+1}\left(\frac{\ln(u)}{n+1} - \frac{1}{(n+1)^2}\right) + C$

128. $\int u^m \ln^n(u)\, du = \frac{u^{m+1}}{m+1}\ln^n(u) - \frac{n}{m+1}\int u^m \ln^{n-1}(u)\, du$

129. $\int e^{au}\ln(u)\, du = \frac{e^{au}\ln(u)}{a} - \frac{1}{a}\int \frac{e^{au}}{u}\, du$

130. $\int \frac{du}{u\ln(u)} = \ln(\ln(u)) + C$

Trigonometrik İfadeler

131. $\int \sin u\, du = -\cos u + C$

132. $\int \cos u\, du = \sin u + C$

133. $\int \tan u\, du = -\ln(\cos u) + C = \ln(\sec u) + C$

134. $\int \cot u\, du = \ln(\sin u) + C$

135. $\int \sec u\, du = \int \frac{du}{\cos u} = \ln(\sec u + \tan u) + C = \ln\left(\tan\left(\frac{u}{2} + \frac{\pi}{4}\right)\right) + C$

136. $\int \mathrm{cosec}\, u\, du = \int \frac{du}{\sin u} = \ln(\mathrm{cosec}\, u - \cot u) + C = \ln\left(\tan\frac{u}{2}\right) + C$

137. $\int \sec^2 u\, du = \tan u + C$

138. $\int \mathrm{cosec}^2 u\, du = -\cot u + C$

139. $\int \sec u \tan u\, du = \sec u + C$

140. $\int \operatorname{cosec} u \cot u \, du = -\operatorname{cosec} u + C$

141. $\int \sin^2 u \, du = \frac{u}{2} - \frac{1}{4}\sin 2u + C$

142. $\int \cos^2 u \, du = \frac{u}{2} + \frac{1}{4}\sin 2u + C$

143. $\int \cos^n u \sin u \, du = -\frac{\sin^{n+1}}{n+1} + C$

144. $\int \sin^n u \cos u \, du = \frac{\sin^{n+1} u}{n+1} + C$

145. $\int \sin mu \sin nu \, du = -\frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C$

146. $\int \cos mu \cos nu \, du = \frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C$

147. $\int \sin mu + \cos nu \, du = -\frac{\cos(m+n)u}{2(m+n)} - \frac{\cos(m-n)u}{2(m-n)} + C$

148. $\int \frac{du}{1+\cos a \cos u} = 2\operatorname{cosec} a \arctan\left(\tan\frac{a}{2}\tan\frac{u}{2}\right) + C$

149. $\int \frac{du}{\cos a + \cos u} = \operatorname{cosec} a \ln\left(\frac{1+\tan\frac{a}{2}\tan\frac{u}{2}}{1-\tan\frac{a}{2}\tan\frac{u}{s}}\right) + C = 2\operatorname{cosec} a \operatorname{arctanh}\left(\tan\frac{a}{2}\tan\frac{u}{2}\right) + C$

öyle ki $\tan^2\frac{u}{2} < \cot^2\frac{a}{2}$

150. $\int \frac{du}{1+\cos a \sin u} = 2\operatorname{cosec} a \arctan\left(\operatorname{cosec} a \tan\frac{u}{2} + \cot a\right) + C$

151. $\int \frac{du}{\cos a + \sin u} = \operatorname{cosec} a \ln\left(\frac{\tan a - \tan\frac{u}{2} - \sec a}{\tan a + \tan\frac{u}{2} + \sec a}\right) + C = -2\operatorname{cosec} a \operatorname{arctanh}\left(\cot a \tan\frac{u}{2} + \operatorname{cosec} a\right) + C$

öyle ki $\left(\cot a \tan\frac{u}{2} + \operatorname{cosec} a\right)^2 < 1$

152. $\int \frac{du}{a^2\cos^2 u + b^2 \sin^2 u} = \frac{1}{ab}\arctan\left(\frac{b\tan u}{a}\right) + C$

153. $\int e^{au} \sin nu \, du = \frac{e^{au}(a\sin nu - n\cos nu)}{a^2+n^2} + C$

154. $\int e^{au} \cos nu \, du = \frac{e^{au}(n\sin nu + a\cos nu)}{a^2+n^2} + C$

155. $\int u \sin u \, du = \sin u - u\cos u + C$

156. $\int u \cos u \, du = \cos u + u\sin u + C$

Trigonometrik ifadelerin tümevarımla integrali

157. $\int \sin^n u \, du = -\frac{\sin^{n-1} u \cos u}{n} + \frac{n-1}{n}\int \sin^{n-2} du$

158. $\int \cos^n u\, du = \frac{\cos^{n-1} u \sin u}{n} + \frac{n-1}{n} \int \cos^{n-2} u\, du$

159. $\int \frac{du}{\sin^n u} = -\frac{\cos u}{(n-1)\sin^{n-1} u} + \frac{n-2}{n-1} \int \frac{du}{\sin^{n-2} u}$

160. $\int \frac{du}{\cos^n u} = \frac{\sin u}{(n-1)\cos^{n-1} u} + \frac{n-2}{n-1} \int \frac{du}{\cos^{n-2} u}$

161. $\int \cos^m u \sin^n u\, du = \frac{\cos^{m-1} u \sin^{n+1} u}{m+n} + \frac{m-1}{m+n} \int \cos^{m-2} u \sin^n u\, du$

162. $\int \cos^m u \sin^n u\, du = -\frac{\sin^{n-1} u \cos^{m+1} u}{m+n} + \frac{n-1}{m+n} \int \cos^m u \sin^{n-2} u\, du$

163. $\int \frac{du}{\cos^m u \sin^n u} = \frac{1}{(m-1)\sin^{n-1} u \cos^{m-1} u} + \frac{m+n-2}{m-1} \int \frac{du}{\cos^{m-2} u \sin^n u}$

164. $\int \frac{du}{\cos^m u \sin^n u} = -\frac{1}{(n-1)\sin^{n-1} u \cos^{m-1} u} + \frac{m+n-2}{n-1} \int \frac{du}{\cos^m u \sin^{n-2} u}$

165. $\int \frac{\cos^m u\, du}{\sin^n u} = -\frac{\cos^{m+1} u}{(n-1)\sin^{n-1} u} - \frac{m-n+2}{n-1} \int \frac{\cos^m u\, du}{\sin^{n-2} u}$

166. $\int \frac{\cos^m u\, du}{\sin^n u} = \frac{\cos^{m-1} u}{(m-n)\sin^{n-1} u} + \frac{m-1}{m-n} \int \frac{\cos^{m-2} u\, du}{\sin^u u}$

167. $\int \frac{\sin^n u\, du}{\cos^m u} = \frac{\sin^{n+1} u}{(m-1)\cos^{m-1} u} - \frac{n-m+2}{m-1} \int \frac{\sin^n u\, du}{\cos^{m-2} u}$

168. $\int \frac{\sin^n u\, du}{\cos^m u} = -\frac{\sin^{n-1} u}{(n-m)\cos^{m-1} u} + \frac{n-1}{n-m} \int \frac{\sin^{n-2} u\, du}{\cos^m u}$

169. $\int \tan^n u\, du = \frac{\tan^{n-1} u}{n-1} - \int \tan^{n-2} u\, du$

170. $\int \cot^n u\, du = -\frac{\cot^{n-1} u}{n-1} - \int \cot^{n-2} u\, du$

171. $\int e^{au} \cos^n u\, du = \frac{e^{au} \cos^{n-1} u (a\cos u + n\sin u)}{a^2+n^2} + \frac{n(n-1)}{a^2+n^2} \int e^{au} \cos^{n-2} u\, du$

172. $\int e^{au} \sin^n u\, du = \frac{e^{au} \sin^{n-1} u (a\sin u - n\cos u)}{a^2+n^2} + \frac{n(n-1)}{a^2+n^2} \int e^{au} \sin^{n-2} u\, du$

173. $\int u^m \cos au\, du = \frac{u^{m-1}}{a^2}(au\sin au + m\cos au) - \frac{m(m-1)}{a^2} \int u^{m-2} \cos au\, du$

174. $\int u^m \sin au\, du = \frac{u^{m-1}}{a^2}(m\sin au - au\cos au) - \frac{m(m-1)}{a^2} \int u^{m-2} \sin au\, du$

Ters trigonometrik ifadeler

175. $\int \arcsin u\, du = u \arcsin u + \sqrt{1-u^2} + C$

176. $\int \arccos u\, du = u \arccos u - \sqrt{1-u^2} + C$

177. $\int \arctan u\, du = u \arctan u - \ln\left(\sqrt{1+u^2}\right) + C$

178. $\int \operatorname{arccot} u\, du = u \operatorname{arccot} u + \ln\left(\sqrt{1+u^2}\right) + C$

179. $\int \operatorname{arcsec} u\, du = u \operatorname{arcsec} u - \ln\left(u + \sqrt{u^2 - 1}\right) + C = u \operatorname{arcsec} u - \arg\cosh u + C$

180. $\int \operatorname{arccosec} u\, du = u \operatorname{arccosec} u + \ln\left(u + \sqrt{u^2 - 1}\right) + C = u \operatorname{arccosec} u + \arg\cosh u + C$

Hiperbolik fonksiyonlar

181. $\int \sinh u\, du = \cosh u + C$

182. $\int \cosh u\, du = \sinh u + C$

183. $\int \tanh u\, du = \ln(\cosh u) + C$

184. $\int \coth u\, du = \ln(\sinh u) + C$

185. $\int \operatorname{sech} u\, du = \arctan(\sinh u) + C = \operatorname{gd} u + C$

186. $\int \operatorname{cosec} u\, du = \ln\left(\tanh \frac{u}{2}\right) + C$

187. $\int \operatorname{sech}^2 u\, du = \tanh u + C$

188. $\int \operatorname{cosech}^2 u\, du = -\coth u + C$

189. $\int \operatorname{sech} u \tanh u\, du = -\operatorname{sech} u + C$

190. $\int \operatorname{cosech} u \coth u\, du = -\operatorname{cosech} u + C$

191. $\int \sinh^2 u\, du = \frac{1}{4} \sinh 2u - \frac{u}{2} + C$

192. $\int \cosh^2 u\, du = \frac{1}{4} \sinh 2u + \frac{u}{2} + C$

193. $\int \tanh^2 u\, du = u - \tanh + C$

194. $\int \coth^2 u\, du = u - \coth u + C$

195. $\int u \sinh u\, du = u \cosh u - \sinh u + C$

196. $\int u \cosh u\, du = u \sinh u - \cosh u + C$

197. $\int \arg\sinh u\, du = u \arg\sinh u - \sqrt{1 + u^2} + C$

198. $\int \arg\cosh u\, du = u \arg\cosh u - \sqrt{u^2 - 1} + C$

199. $\int \arg\tanh u\, du = u \arg\tanh u + \frac{1}{2} \ln\left(1 - u^2\right) + C$

200. $\int \arg\coth u\, du = u \arg\coth u + \frac{1}{2} \ln\left(1 - u^2\right) + C$

201. $\int \arg\operatorname{sech} u\, du = u \arg\operatorname{sech} u + \operatorname{gd}(\arg\tanh u) + C = u \arg\operatorname{sech} u + \arcsin u + C$

202. $\int \arg\operatorname{cosech} u\, du = u \arg\operatorname{cosech} u + \arg\sinh u + C$

203. $\int \sinh mu \sinh u\, du = \frac{\sinh(m+n)u}{2(m+n)} - \frac{\sinh(m-n)u}{2(m-n)} + C$

204. $\int \cosh mu \cosh nu\, du = \frac{\sinh(m+n)u}{2(m+n)} + \frac{\sinh(m-n)u}{2(m-n)} + C$

205. $\int \sinh mu \cosh nu\, du = \frac{\cosh(m+n)u}{2(m+n)} + \frac{\cosh(m-n)u}{2(m-n)} + C$

206. $\int \frac{du}{\cosh a + \cosh u} = 2 \operatorname{cosech} a \cdot \arg\tanh\left(\tanh \frac{u}{2} \tanh \frac{a}{2}\right) + C$

207. $\int \frac{du}{\cos a + \cosh u} = 2 \operatorname{cosec} a \cdot \arg\tan\left(\tanh \frac{u}{2} \tan \frac{a}{2}\right) + C$

208. $\int \frac{du}{1 + \cos a \cosh u} = 2 \operatorname{cosec} a \cdot \arg\tanh\left(\tanh \frac{u}{2} \tan \frac{a}{2}\right) + C$ eğer $\tan^2 \frac{u}{2} < \cot^2 \frac{a}{2}$ ise

209. $\int e^{au} \sinh nu\, du = \frac{e^{au}(a \sinh nu - n \cosh nu)}{a^2 - n^2} + C$

210. $\int e^{au} \cosh nu\, du = \frac{e^{au}(a \cosh nu - n \sinh nu)}{a^2 - n^2} + C$

Kaynakça

Granville, Smith ve Longley, **Eléments de Calcul Différentiel et Intégral, Vubert 1970.**